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Strategy to address two-dimensional pointwise concave contact problems

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Contact detection is a very important computational task for different fields, from computer graphics to computational contact mechanics. In the last, traditionally, one of the curves/surfaces is discretized into a set of points that are paired with their projections onto the other curve/surface, a procedure called Closest Point Projection (CPP) [1]. Problems of non-uniqueness arise when dealing with concave bodies [2].

In the context of pointwise contact, usually a valid approximation for non-conformal contact with small deformations, an approach not requiring prior discretization of any boundary was proposed for curve-to-curve [3] and surface-to-surface [4] contacts. In the curve-to-curve case, one searches for a pair of points that satisfies orthogonality relations necessary for the minimization of the objective function

$$f(x_1, x_2) = \frac{1}{2}\left(\Gamma_A(x_1) - \Gamma_B(x_2)\right) \cdot \left(\Gamma_A(x_1) - \Gamma_B(x_2)\right) = \frac{1}{2}\left(d(x_1, x_2)\right)^2, \quad (1)$$

which gives the squared distance between material points of two planar curves Γ_A and Γ_B, parameterized by x_1 and x_2 coordinates, respectively.

Such search for pairs of contact points is referred to as Local Contact Problem (LCP). A peculiarity of the approach is that for intersecting boundaries, which is the case when a penalty-like method is employed to enforce contact, the contact pairs are not actual minimizers of f, but saddle points [5]. It is possible to convert the LCP into a minimization problem [6], enabling the use of more robust optimization methods to solve the LCP. However, the approach is restricted to convex bodies.

In this work, we investigate properties of the objective function in equation (1) when the bodies do not intersect each other. It is supposed that a barrier-like method is used to enforce contact while preventing intersections. The bodies may contain concavities, but it is supposed that all curves are composed of strictly convex segments. The convex segment is said to be a concave boundary if the outward normal of the body $\boldsymbol{n}_{\text{ext}}$ points to the same direction of the normal of the segment $\boldsymbol{n}_\Gamma$, and it is said to be a convex boundary otherwise (Figure 1).

(a) (b)

Figure 1: Convex segments representing (a) convex boundary (b) concave boundary.

We first show that, for non-intersecting configurations, only minimizers of f can effectively contribute to contact. Then, we proceed to a discussion of uniqueness of contact pairs in two cases (Figure 2). The first one is the contact of convex bodies. We prove by geometric considerations that, even though the minimizer of f is not necessarily unique, the contact pair is unique.

(a) (b)

Figure 2: Contact cases for (a) convex boundaries (b) convex and concave boundaries.

The second case is the contact between convex and concave boundaries. A univariate restriction of f is obtained by projecting the concave boundary onto the convex boundary. The properties of the restricted function are deeply related to geometric properties of the curves. The relations are encapsulated into the function

$$g(x_1) = R_B\big(x_2(x_1)\big) - R_A(x_1) - d(x_1), \tag{2}$$

where $R_A(x_1)$ is the curvature radius of the convex boundary at point x_1, $R_B\big(x_2(x_1)\big)$ is the curvature radius of the concave boundary at point x_2 with projection x_1, and $d(x_1)$ is the distance between the two points. The function g can be used to split the curves into regions, thus isolating the minimizers of f. With that, all plausible contact points are expected to be found.

Examples will be presented to demonstrate the behavior of the proposed strategy for detecting pointwise contact between general curves.

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